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# Improving Recommendation Diversity based on Latent Factor Model

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***ABSTRACT:*** *Through the years, Recommender Systems have been trained towards maximizing recommendation accuracy in order to propose good recommendation. However, focusing on improving accuracy arises an overspecialization problem.To overcome this problem, the need of diversity in recommendation while maintaining accuracy has become crucial.Thereby, we propose a User-based Collaborative filtering approach based on a Non-negative matrix factorization with the ultimate objective of enhancing users' satisfaction by improving the trade-off between accuracy and diversity. Our proposed approach finds the latent factors from the existing data using Non-negative matrix factorization algorithm to regroup users in clusters. Using these clusters, we form the neighborhood set based on the level of diversity of each user to make it as input for the User-based Collaborative filtering algorithm. Additionally, we suggest an integration of a re-ranking algorithm on the recommended list. For the evaluation part, we apply the 5-fold cross validation technique to real datasets. Accordingly, the experimental results have proved the performance and robustness of the new approach on balancing accuracy and diversity in comparison with the benchmark algorithms.*

***KEYWORDS:*** *Recommender Systems, Diversity, User-based Collaborative filtering, Non-negative matrix factorization*

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## I. INTRODUCTION

In recent years, the growth of the World Wide Web has caused a potential challenge of information overload. Therefore, the difficulty for users to quickly find pertinent information has been raised, given the tremendous amount of data. Concerning this issue, Recommender Systems (RSs) have become an efficient personalized solution to quickly find relevant contents and items[1],[2].They are nowadays essential part of many service-oriented applications [3]like movies, news, travel guides, restaurants and particularly E-commerce sites, due to their ability to improve company’s profitability by offering the best-matching items for the potential customer who will be most likely to make a purchase [4], [5].

## II. BACKGROUND

In this section, we present the different concepts related to our work. We begin by describing the collaborative filtering approach, then introduce the notion of Non-negative matrix factorization (NMF).

### 2.1. Collaborative Filtering Techniques

First, Collaborative filtering (CF) is one of the most popular and effective approach to construct recommendation systems. Beyond academic interest, it has been extensively applied in various domains such as E-Commerce industry e.g. Amazon, Ebay, and Netflix websites [3],[6]. The fundamental idea of collaborative filtering is based on social behavior used by human for centuries; it represents the desire of sharing ideas and opinions with others. More specifically, if two users have similar liking products in the past, then they will certainly share the same opinions about other products in the future as well [7]. In a typical scenario, the data used can be represented as a list of n users.$\{u_1, u_2, \ldots, u_n\}$and a list of m items$\{i_1, i_2, \ldots, i_m\}$. Each user is interested in a list of items for which he expressed his opinion about either implicitly, such as purchase records, clicks or visits, or explicitly, on a numerical five-star scale, where one and two stars represent negative ratings, three stars represent ambivalence, while four and five stars represent positive ratings. The correspondence between users and items are usually represented in the form of user-item ratings matrix:

$$R \in R^{|u|\times|i|} \tag{1}$$

Since users do not usually give their preferences about certain items, the R matrix has generally a value of sparsity larger than 99% in commercial systems [8]. Table 1 illustrates an example of user-item rating matrix, representing 5 users' interest. Users are denoted as $u_1$to$u_5$, rating on a $1-5$ scale, on a list of 10 items (denoted as $i_1$to$i_{10}$). The blanks represent the situation where the user has not rated the item. The goal of recommendation system algorithm is to estimate the missing entries in the user-item rating matrix in order to recommend items to users based on the highest predicted values.
Based on the usage of CF algorithms of the user-item matrix, [9] has classified the CF approaches into two main classes:

1. Memory-based CF methodsuse the entire user-item matrix in order to make recommendations by finding similar users or, alternatively, similar items to the queried user or item.

This specific class itselfis divided in two categories:

a. User based collaborative filtering algorithms (UBCF) [8],[10]. UBCF identify the nearest neighbors to the target user based on similarities between users and generate a prediction by averaging ratings of these neighbors.
b. Item-based CF algorithms starts to compute the similarity between a target item and the set of items the target user has already rated and then select the most similar items to the target item. The prediction is then calculated by averaging the target users' ratings on these similar items.

2. Model-based CF methods[9]: make recommendations by training a model based on the ratings matrix. Popular methods used for constructing the model are: latent factor models such as Matrix Factorization [11] and Singular Value Decomposition [12], Markov-based models [13], Bayesian methods [14] and sparse linear methods [15].

**Table 1:** User – Item rating matrix

| | $i_1$ | $i_2$ | $i_3$ | $i_4$ | $i_5$ | $i_6$ | $i_7$ | $i_8$ | $i_9$ | $i_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $u_1$ | 1 | | 4 | | | 2 | 2 | | 3 | |
| $u_2$ | | 3 | 1 | | 2 | | | 5 | 4 | |
| $u_3$ | 4 | | 2 | | | 5 | | 5 | | 5 |
| $u_4$ | 1 | | 4 | | 2 | 2 | 2 | | 2 | |

In real-world applications, Memory-based techniques are quite successful because of their simplistic approach and ease to implementation. However, despite their widespread use, they have some drawbacks such as data sparsity and data scalability issues that could lead to an inaccurate performance and a very poor scalability of the recommendation system [16], [17]. To overcome these weaknesses, we explored the robustness of Non-Negative Matrix Factorization (NMF)which will be described in more detail in the next subsection. One advantage of using this technique is that instead of having a high dimensional matrix containing abundant number of missing values we will be dealing with a much smaller matrix in lower-dimensional space and also comparing similarity on the resulting matrix is much more scalable especially in dealing with large sparse datasets in the one hand.
In the other hand, the Collaborative filtering recommendation techniques often suffer from the problem of bias towards popular items. These techniques only consider products that are similar to those that the neighborhood of the user had evaluated in the past [7]. As consequences, if users' profiles are very similar to each other, the recommended items will be also of great similarity to each other and already known by the user [18], [19]. Thereby, users who are not very similar to each other are more likely to increase the diversity of the recommended items. Based in these assumptions, the proposed approach chooses $k-diverse$ neighbors, rather than the $k$most similar neighbors. Indeed, an important issue affecting the recommendation results is popularity of the items [20]. Most CF algorithms that are designed to provide accurate results often recommend popular items, although users might be interested in non-popular and novel ones. Furthermore, the number of popular items is often much less than others. This usually causes similar recommendations for different users, which is not generally desired. Recommendations should be diverse and novel to satisfy the users [21].

### 2.2. Non-Negative Matrix Factorization (NMF)

The precept of the factorization $X = UV'$of a matrix is widely used in principal component analysis which uses the Singular Value Decomposition of the matrix $X$ $(SVD)$ to construct orthogonal factors two by two. The NMF is a dimension reduction technique adapted to hollow matrices containing positive data, for example occurrences or counts of words, failures ... The method is therefore more suited to certain situations than the $SVD$. Non-orthogonal factors are used as a basis for unsupervised or prior modelling classification for supervised learning.

The following description of the NMF method is not meant to be exhaustive; Let $X$ be a matrix $(n \times p)$ containing only non-negative values and without a row or column containing only 0; $r$ is a chosen integer relatively small comparing to $n$ and $p$.
The non-negative factorization of the matrix $X$ is the search for two matrices $W_{n\times r}$ and $H_{r\times p}$ containing only positive or zero values and whose product approaches X.

$$X \approx WH \tag{2}$$

The choice of factorization rank $r << min\,(n,p)$ ensures a drastic reduction of dimension and therefore parsimonious representations. The factorization is solved by the search for a local optimum of the optimization problem:

$$\min_{W,H\geq 0}\,[L(X,WH) + P(W,H)] \tag{3}$$

$L$ is a loss function measuring the approximation quality and $P$an optional penalization function; $L$ is generally either a least squares criterion ($LS$or Frobenius norm of the matrices or "trace norm"), or the $Kullback-Leibler$ divergence ($KL$); $P$ is an optional regularization penalty used to force the desired properties of the matrices $W$ and$H$.

$$LS: L(A,B) = tr\left((A-B)(A-B)'\right) = \sum_{i,j}\left(a_{i,j}-b_{i,j}\right)^2 \tag{4}$$

$$KL : L(A,B) = KL(A||B) = \sum_{i,j} a_{i,j}\, log\left(\frac{a_{i,j}}{b_{i,j}}\right) - a_{i,j} + b_{i,j} \tag{5}$$

In the NMF library of $R$, the variables (features) are in line and the individuals/ samples are in columns. This does not matter when the least squares criterion is used ($LS$), the resolution is invariant by transposition but makes sense with the $Kullback - Leibler$ divergence which introduces asymmetry between rows and columns.
P.S. Not only is the solution local because the objective function is not convex in $W$ and $H$ but in addition the solution is not unique. Any non-negative and invertible $D_{r\times r}$ matrix provides equivalent solutions in terms of fit:

$$X \approx WDD^{-1}H \tag{6}$$

Once the factorization is built it is then easy to use these matrices $W$ and $H$ to build classifications ($CAH, k-means$), representations as Principal Component Analysis (PCA), and forecasts using one of the many methods of learning.

## III. RELATED WORK

Before Generally, diversification techniques in Recommender System literature distinguishes two lines of research depending on the level at which the diversification is integrated, namely post-filtering approaches and diversity modelling [22]. The post-filtering methods consists in using heuristic strategy to re-rank the set of recommendations, which mainly contains two steps: application of a standard RS to get a set of accurate candidate items, and then selecting the $Top - N$ candidate items by maximizing a specified diversification criterion. Instead, the diversity modelling strategy, which is the focus of this paper, aims to enhance recommendation algorithms by integrating diversification analysis prior to the ranking procedure of the RS, so that the accuracy as well as the diversity are considered at the same time. [23] provided an overview of diversification methods and an extensive survey on evaluation metrics.
The first approach that introduced the diversification in the recommendation system was [24]. The authors propose a greedy-based selection algorithm based on pairwise dissimilarities to characterize the diversity property of the list in order to achieve better trade-off between accuracy and diversity. Another work of [25] treated the topic of diversification for book recommendation, where the re-ranking is based on a genre taxonomy-based metric. [25] were the first to provide an analysis study of how their method affected real user satisfaction. The authors found that the satisfaction of real user improves even though increasing diversity influences negatively accuracy. Some works proposed using the semantic taxonomy information to better model

the items utility in the ranked list as [26], [27], [28] and [29]. [27] proposed a binomial framework for greedy re-ranking approach based on genre diversity. The resulting method thus aims to maximize coverage of genres considering the user preferences while the frequency of each genre should be represented equally and finally considers the screen size limitation to offer recommendations. [29] proposed an adaptive multi-attribute diversification procedure to re-rank the recommendation list previously provided by a generic recommendation algorithm. The re-ranking procedure is based on the user profile classification into quadrants using fuzzy classification approach where for each attribute describing the item, the user profile will belong to each quadrant with a certain degree. Some other works on post-filtering approach for diversity has focused on designing more advanced objective functions that combine item relevance and diversity. [30] modelled the process of recommendation as a multi-objective optimization problem. Firstly, the item-based collaborative filtering methods are employed to generate a candidate items which will be used to optimize the objective function of diversity by exploring the genre information of each candidate item and the multi-objective evolutionary algorithm based on decomposition. Also, [31] defined a greedy MAP inference for DPP-based approach for an adjustable trade-off between relevance and diversity. Using a known item-based recommendation algorithm as input, they assign higher probability to sets of items that are diverse from each other. While other research proposed to define the diversity metric based on clustering, [32] proposed two diversification methods using ontology-based semantic distance to cluster items by applying the k-means algorithm. The Cluster Random method selects the best item in the cluster chose randomly according to the users' preferences and it is moved to the TOP-N recommended list. In contrary, the Cluster Quadratic method, in each of the N iterations, the best item is selected from the first item in each cluster based on the max value of a calculated score that balances the item accuracy and diversity.

An obvious advantage of the post-filtering techniques described in the previous paragraph is the ease of deployment in recommendation systems already implemented where the diversification process is incorporated after the generation of a candidate items by the existing recommendation algorithms. However, it depends directly on the generated candidate items list which must be already diversified. This problem is circumvented by the other class of approaches, diversity modelling approaches, which allows a better control of diversity. There are several works based on this latter. As in [33], [34], where they used user profile partitioning diversification techniques. Basically, it consists of partitioning the user profile in the form of clusters or sub-profiles, then the recommendations are generated by treating each sub-profile as being an independent user profile. In addition, some works focused on the latent factor-based techniques to produce more diverse recommendations. For instance, [35] used the Matrix Factorization method based on category features of items. The score of each candidate item is calculated by summing the relevancy part and discounted diversification part. The first part equal to the product of user interest vector and item attribute vector, and the discounted product of user and category features corresponds to the second part. As well as [36], the authors used the Matrix Factorization approach to optimize an award function that uses the interest of items formulated as novelty to increase the diversity of the recommended list. Besides, [37] started by estimating the topic distributions of users and items using the latent Dirichlet Allocation method on the users' ratings. Then, they proposed two diversification methods according to the nature of diversification: proportionally or marginally. In Social Recommendation context, [38] applied the theory of social curiosity along with the user preferences measured by the Matrix Factorization method to construct the user interests. The recommendation list corresponds to the top ranked items based on user interests. While [39] has combined a social recommendation and label recommendation to improve the diversity of recommended items. The proposed approach is based on hybrid label recommendation based on Matrix Factorization. Some other works address some clustering techniques to improve recommendation diversity. [40] used a pre-filtering approach based on K-Medoids clustering algorithm. They grouped users with the same degree of preference in diversity using the distribution of categories associated with items in their profiles. Then, they applied a standard user-based collaborative filtering algorithm for each segment to get recommendations. Also, a user-centric conceptual framework, proposed by [41], was developed based on four proprieties inspired from Stirling's definition [42]: global coverage (variety of categories), novelty and local coverage (unfamiliar and familiar categories with current preferences), redundancy (the number of items correspond to the same category). [41] modelled the user profile based on the item categories using LSA-based methodology. Then, the clusters of similar users are formed to select the distant neighborhood belonging to the same cluster which will be incorporated into a recommendation filtering process. In parallel, [43] defined a third dimension, category of item, to be added into the traditional user/product recommendation relationship to form a triangle model. The proposed approach clusters user behaviours by combining the k-mean, Markov chain and collaborative filtering algorithm to make accurate and diverse recommendations. The area of the constructed triangle model of a product is calculated using Herons formula. Then the recommendations are made according to the larger triangular areas.

## IV. PROPOSED APPROACH

Different from the above approaches, we propose an improved User-based Collaborative filtering approach based on Non-negative matrix factorization (NMF-CF), depending on the basic CF procedures but changing the set of selected neighbors in order to provide more accurate and diverse recommendations without increasing the complexity of the algorithm. Moreover, to customize the diversity for each user, we have integrated an algorithm to estimate the level of diversity for each user. The proposed approach can be divided into three main stages as illustrated in Figure 1. First, the user-item matrix is used as input to the NMF algorithm in order to form the Neighborhood set. Next, this Neighborhood set is used to provide the top-N candidate items using the User-Based Collaborative Filtering algorithm. Finally, the third step can significantly maximize satisfaction of users by re-ranking the candidate items to recommend the Top-N accurate and diverse items to the user.

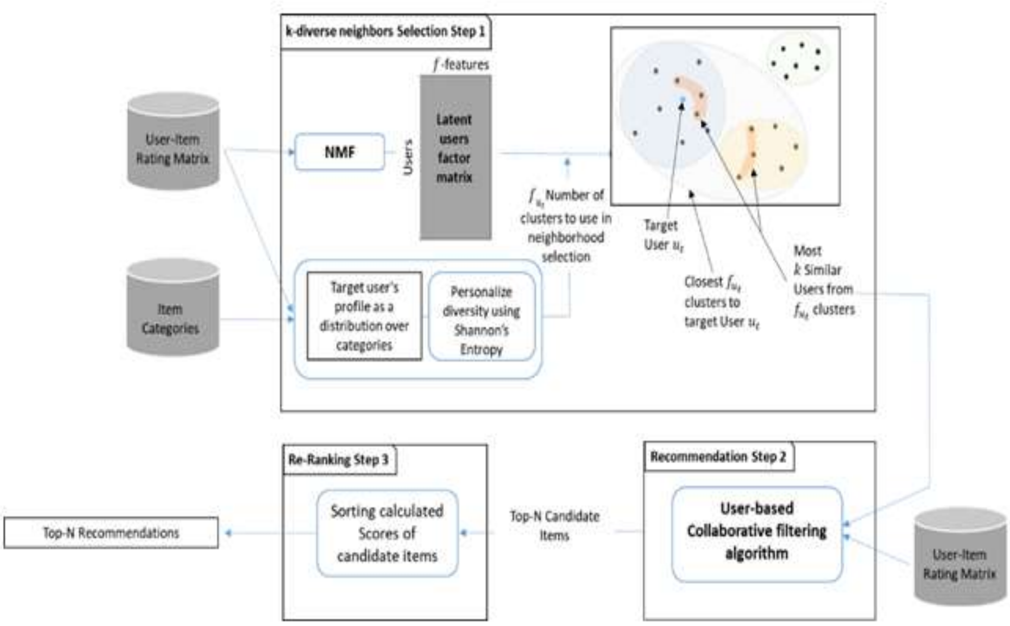


**Figure 1:** A schematic representation of the Proposed Approach

### 4.1. k-diverse neighbors Selection Step based on NMF:

In this section, we will describe the detailed steps of the first phase of the proposed approach and provide algorithm information in Algorithm 1. The goal of this step is to select k-diverse neighbors of the target user to be utilized by the User-based Collaborative filtering algorithm in the next step. This set will allow us to recommend diverse items without influencing the accuracy. So, to meet this requirement, we started by applying the NMF algorithm on the User-item ratings matrix in order to learn the latent features of users and group the users according to these latent features.

Let $\boldsymbol{U_{n\times f}}$ be latent users factor matrix, with each row vector represent f-dimensional user-specific latent feature vector. We consider each latent factor as a cluster. Accordingly, we will generate $\boldsymbol{C_f}$clusters of users such that each user of a cluster $\boldsymbol{C_i}$ has a maximum extent of interest in the corresponding latent factor $i$.

$$C_i = \{u \in U_{n\times f} \,|\, argmax_{k\in f} U(k)_{n\times f} = i\} \tag{7}$$

After constructing the different clusters comes the neighborhood selection phase. In this phase, if we take the users who belong to the same cluster of the target user, we will have items that will be relevant to the user but not diverse since the interest in items of his neighborhood is similar to that of the target user. So, to ensure an acceptable diversity level for the target user, we propose to build the neighborhood set based on several clusters and not just the cluster where the target user belongs. In order to do that, two constrained questions must be answered:

- Are we going to use all the $\boldsymbol{C_f}$clusters we build to choose the diverse neighbors or just some of them?
- Which users to select from the clusters once we have chosen them?

For the first constraint, the choice of the clusters to use depends necessarily on each user and their openness to experience [44]. Openness to experience measures the level of curiosity of a person, his interest in new experiences and ideas which shows us that each user has his own interest in diversity. In other words, users who are more open to the experience would like to receive diverse recommendations as opposed to those who are less open to experience, they prefer to receive recommendations that are similar to previously appreciated items. In this work, we have chosen the definition of diversity of [27], which defines the notion of diversity using the category information for items (e.g., genres in movies or food type in restaurants). So that, the recommended list is considered diversified when it covered several categories while avoiding redundancy. Using the aforementioned characteristic of items, we can represent the user's profile as a distribution over categories [34]. Assuming that each item belongs to one or more categories in $\boldsymbol{G}$, we define the probability that a user $\boldsymbol{u}$ is interested in a category $\boldsymbol{g_l}$ by:

$$P(g_l|u) = \frac{\sum_{i \in I_u} r_{u,i} \times P(i \in g_l)}{\sum_{j \in |G|} \sum_{i \in I_u} r_{u,i} \times P(i \in g_j)} \tag{8}$$

Where $\boldsymbol{I_u}$ is a set of all items rated by the user $\boldsymbol{u}$, $\boldsymbol{r_{u,i}}$ is the rating assigned by the user $\boldsymbol{u}$ to the item $\boldsymbol{i}$ and $\boldsymbol{P(i \in g_l)}$ is the probability that an item$\boldsymbol{i}$ belongs to the $\boldsymbol{category g_l}$ and it is defined as a binary value as follows:

$$P(i \in g_l) = \begin{cases} 1 & if\ item\ i\ belongs\ to\ category\ g_l, \\ 0 & otherwise. \end{cases} \tag{9}$$

In order to measure the openness or the level of diversity for each user, we used the Shannon Entropy Diversity Metric formulated as follows [45]:

$$H(u) = -\sum_{l=1}^{|G|} P(g_l|u) log_2(P(g_l|u)) \tag{10}$$

We suppose that the user who has the max value of the Shannon's Entropy has the highest interest in diversity. Thereby, he will be interested in the most diversified neighbors' opinions, i.e. the selection of neighbour's users must be made on all clusters. Based on this supposition and defining the max value of the Shannon's Entropy over all users as the highest level of diversity, the number of clusters to use to select the neighborhood of a user $u$ is calculated by the following formula:

$$f_u = (f \times H(u))/H_{max} \tag{11}$$

Where $\boldsymbol{f}$ is the Number of latent features, $\boldsymbol{H(u)}$ is the Shannon's entropy value of the user $\boldsymbol{u}$ and $\boldsymbol{H_{max}}$ is the maximum Shannon's entropy value over all users. Consequently, the set of the top $\boldsymbol{f_u}$ groups $C_{\boldsymbol{f_u}}$ will be selected from the latent factor vector $\boldsymbol{U_{u \times f}}$ of the target user $\boldsymbol{u}$ sorted in descending order.

For the second constraint, we must choose a set of neighbors to select from each previously defined clusters $C_{\boldsymbol{f_u}}$. In order to not lose in accuracy, we choose the k-diverse neighbors who are the most similar to the target user $\boldsymbol{u}$ in each cluster of $C_{\boldsymbol{f_u}}$. The Pearson correlation coefficient is popularly used as a similarity measure between a target user $\boldsymbol{u}$ and a user $\boldsymbol{v}$:

$$similarity(u,v) = \frac{\sum_{i \in I_u \cap I_v} (r_{u,i} - \overline{r_u})(r_{v,i} - \overline{r_v})}{\sqrt{\sum_{i \in I_u \cap I_v} (r_{u,i} - \overline{r_u})^2} \sqrt{\sum_{i \in I_u \cap I_v} (r_{v,i} - \overline{r_v})^2}} \tag{12}$$

Where $\boldsymbol{I_u}$ is the set of all items rated by target user $\boldsymbol{u}$, and $\boldsymbol{\overline{r_u}}$ is the average rating of target user $\boldsymbol{u}$. After selecting k-diverse neighbors, we will pass, in the next step, this selection of neighborhood set to the User-based Collaborative filtering algorithm to make recommendation.

Algorithm 1: Selection of k-diverse neighbors of target user using Non-Negative Matrix Factorization algorithm

**Require**: User-item rating matrix,
$f$**:** Number of latent features,
$k$: Number of users in the neighborhood $NU_{u_t}$ of target user $u_t$.
$H(u_t)$: Shannon entropy value of target user $u_t$.
$H_{max}$ : Maximum Shannon's entropy value over all users.

**Ensure**: k-diverse neighbors $NU_{u_t}$ of target user $u_t$
1: Generate latent user factor matrix $U_{n\times f}$ using the Non-Negative Matrix Factorization algorithm which takes as input the User-item rating matrix.
2: **for**$i = 1$ to $f$ do
3: $C_i = \{u \in U_{n\times f} \middle| argmax_{k\in f} U(k)_{n\times f} = i\}$
4: **end for**
5: Number of nearest clusters $f_{u_t} = (k \times H(u_t))/H_{max}$ .
6: Number of users per cluster $l_{u_t}^c = k/f_{u_t}$.
7: $C_{f_u} \leftarrow$ top $f_{u_t}$ clusters from the latent factor vector $U_{u_t\times f}$ of target user $u_t$ sorted in descending order.
8: **for each**$c \in C_{f_u}$
9: $NU_{u_t} \leftarrow NU_{u_t} \cup most\ l_{u_t}^c\ similar\ users\ to\ the\ target\ user\ u_t inclusterc$
10: **end for**
11: **return**$NU_{u_t}$

**4.2. Recommendation Step using UBCF**
In the second stage, we propose to improve the traditional user-based CF technique (UBCF) to recommend diverse items to users using the k-diverse neighbors set selected in the previous step. The traditional UBCF consists of the following phases:

Phase 1: Neighborhood selection. In order to select the k-nearest neighbors' users for a target user, we calculate similarities between users using a certain similarity function. Pearson correlation coefficient can be used to calculate the similarity measure between users using the formula (12). So, based on the similarity values, the first $K$ most similar users are considered as $k$-nearest neighbors of a target user. A description of the procedure is shown in **Algorithm 2**.

Algorithm 2: The k-nearest neighbors' algorithm for target user $u$

**Input**: User set $U = \{u_1, u_2, \dots, u_m\}$
**Input**: Target user $u$
**Input**: Number $k$ of neighbors
**Output**: $k$-nearest neighbors for user $u$
Foreach $v \in U\backslash\{u\}$ do
$sim_{u,v} = similarity(u, v)$;
select $k$ neighbors $v \neq u$ for $u$ with highest $sim_{u,v}$

Phase 2: Prediction and Recommendation. The predicted rating for a candidate item which have not been rated by the target user $u$, is obtained from an aggregate of the ratings from the user's neighbors who have rated the candidate item. The rating score $p_{u,i}$of a candidate item $i$ for target user $u$ is calculated using the weighted sum approach given by equation (14):

$$p_{u,i} = \alpha \sum_{v\in NU_i} sim_{u,v} \times r_{v,i} \quad (13)$$

Where $NU_i$ is the set of k-nearest users who have rated item $i$, and multiplier $\alpha$ is a normalizing factor.

$$\alpha = \frac{1}{\sum_{v\in NU} sim_{u,v}} \quad (14)$$

According to the predicted rating scores, the highly rated items from the candidate items are recommended to the target user $u$.

**4.3. Re-ranking Step:**
As a result of the previous step, a list of recommended items that are accurate and diverse, but the ranking or the position of this items is important. As a matter of fact, it is possible that if the user doesn't like the first items, it will decrease the probability that he will keep scrolling to see other items. That's why we proposed a $\boldsymbol{3^{rd}}$ step to re-rank the recommended list. Algorithm 3 consists in enhancing satisfaction of user by re-ranking the items according to their scores calculated according to the formula described in equation (20).

Algorithm 3 Re-ranking Algorithm

**Require**: list of candidate items $CI_{u_t}$ of the target user $u_t$ after applying UBCF
**Ensure**: Recommended items set of size $N$ for the target user $u_t$.
1: **for** each candidate item $i \in CI_{u_t}$ do
2: calculate $score_{u_t,i}$ for each candidate item $i$ using equation (20).
3: **End for**
4: Recommend to the target user $u_t$ the top N candidate items having the highest $score_{u_t,i}$.

## V. EXPERIMENTS & RESULTS

### 5.1 Datasets

Since we have defined diversity based on the category information of items, the databases chosen to evaluate the performance of our approach must meet a fundamental requirement: each item should be associated with category information. There are several types of databases for recommender systems that meet this requirement. We conduct our experiment on real Movie Rating Datasets that categorize movies in different genres: MovieLens 100K (ML-100K), MovieLens 1M (ML-1M) datasets [46] and MovieTweetings dataset (MT-750K) [47]. The ML-100K dataset consists of 100,000 ratings for 1682 movies from 943 users and ML-1M dataset provides around 1,000,209 ratings of 3,900 movies made by 6,040 users. Both datasets contain ratings that made on a 1 to 5 scale and each user has rated at least 20 movies. The latest MT-750K dataset contains 742,993 ratings on a 1 to 10 scale contained in tweets posted on Twitter by 55,429 users for 32,348 movies. In order to have a database for the MT-750K dataset with a manageable size in our experiment, we just selected 10,000 users and kept the other information. The dataset statistics are represented in Table 2.

**Table 2**. Datasetstatistics

| | ML-100K | ML-1M | MT-750K |
|---|---|---|---|
| Users | 943 | 6,040 | 10,000 |
| Movies | 1,682 | 3,900 | 32,348 |
| Ratings | 100,000 | 1,000,209 | 130,329 |
| Rating scale | 1-5 | 1-5 | 1-10 |
| Genres | 19 | 18 | 28 |
| Average number of genres | 1.7 | 1.6 | 1.9 |
| Sparsity | 94 % | 95,75 % | 99,99 % |

For each dataset, we perform a 5-fold cross validation for training and test set which contains 80% and 20% of movie ratings respectively. For the ground truth set, to ensure that only relevant items are taking into consideration for a user in the test set, we set the relevant rating threshold as the average rating of that user since each user has his own scale of relevancy of items, i.e. a user gives a score of 3 as the best score contrary to another who gives the score of 5 for the best items.

### 5.2. Baseline methods

In order to evaluate the competitiveness of our approach, we compare it with following baseline methods:

- UBFC: The basic User-Based Collaborative filtering algorithm [16] using the Pearson correlation coefficient as similarity measure.
- NMF: The standard Non-negative matrix factorization optimized by NNLS active-set algorithm [48] where the predicted rating is obtained from a dot product of the two vectors corresponding to latent factor vector of users and item. For the NMF parameters, we kept the suggested parameters in [48].
- S-TDERank: A Standard to Total Diversity Effect Ranking method [49] was proposed to improve diversification. The proposed approach applies in the first place the user-Based collaborative filtering to generate Top-N + S recommendation items. Then, they apply the Total Diversity Effect ranking based on the total diversity effect of each item to generate the Top-N recommendations items. We set the S to 4 in our experiments.

### 5.3. Evaluation Metrics

### 5.3.1 Accuracy

- **Precision ($Prec$)**

In order to measure the accuracy of the results of an RS, it is not enough to measure the accuracy of the individual predictions, but also measure the user agrees with the proposed recommended set. Precision is one of the most common basic information retrieval metrics widely used to measure the quality of recommended set. It is the proportion of relevant recommended items from the total number of recommended items:

$$Prec@N = \frac{1}{|U|}\sum_{u \in U}\frac{|\{i \in L_u | r_{u,i} \geq \overline{r_u}\}|}{N} \tag{15}$$

Where $U$ is a set of users and $L_u$ is the set of $N$ recommendations proposed to user $u$ and taking the average rating $\overline{r_u}$ of $u$ as relevance threshold.

- **Normalized Discounted Cumulative Gain (NDCG)**[50]

NDCG is one of the most popular evaluation measures in Web search [51]. It has two advantages compared to many other measures. First, it allows each retrieved document has graded relevance while most traditional ranking measures only allow binary relevance. Second, NDCG involves a discount function over the rank while many other measures uniformly weight all positions. It is calculated based on the updated cumulative gain (Discounted Cumulative Gain -DCG) measure, and then compared to the ideal classification of relevant items (Ideal Discounted Cumulative Gain - IDCG). In recommendation systems, user ratings can naturally serve as judgments of relevance (gain). The $NDCG_u$ at the $N$-th rank relative to a user u for a list of $N$ recommended items are defined as follows:

$$NDCG_u@N = \frac{DCG_u@N}{IDCG_u@N} \tag{16}$$

Where:

$$DCG_u@N = \frac{\sum_{i=1}^{N} 2^{rel\,(i)}}{log_2(i+1)},\ rel(i) = r_{u,i} - \overline{r_u} + 1 \tag{17}$$

**5.3.2. Diversity**

- **Intent-aware precision** $(\boldsymbol{Prec - IA})$

Several researches have focused on defining an appropriate evaluation measure that can be used to provide information about diversity of recommendation lists. Assume there are $\boldsymbol{m}$ categories of items, Agrawal et al. [52] proposed an intent-aware version of precision. They turned the traditional precision into intent-aware measure by averaging over subtopics. Then, intent-aware precision at the N-th position for a list of $\boldsymbol{N}$ recommended items for a user $\boldsymbol{u}$ is defined as:

$$Prec - IA_u@N = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{N}\sum_{j=1}^{N} J(i, d_j) \tag{18}$$

Where $\boldsymbol{J(i, d_j)}$ = 1 if the $\boldsymbol{d_i}$ includes category$\boldsymbol{i}$; otherwise $\boldsymbol{J(i, d_j)}$ = 0.

- **α-nDCG**

α-nDCG [53] is an extension of DCG,it uses a position-based user model. The measure considers the position at which a document is ranked along with the subtopics contained in the documents. α-nDCG scores a ranking by rewarding newly-found subtopics and penalizing redundant subtopics geometrically, discounting all rewards with a log-harmonic discount function of rank. α is a parameter controlling the severity of redundancy penalization. The $\boldsymbol{\alpha - nDCG}$ at the N-th position relative to a user $\boldsymbol{u}$ for a list of $\boldsymbol{N}$ recommended items are defined by equation (19):

$$\alpha - nDCG_u@N = \frac{1}{\alpha - IDCG_u@N}\sum_{i=1}^{N}\frac{G_i}{log_2(i+1)} \tag{19}$$

With the gain $\boldsymbol{G_i}$ is defined by:

$$G_i = \sum_{l=1}^{m} J(d_i, l)(1-\alpha)^{r_{l,i-1}} \tag{20}$$

Where:

- i-th category (thematic, genre)
- $d_i$ the item ranked in the i-th position on the recommended list;
- $r_{l,i-1}$ the number of items d includes category $l$ positioned before position $i-1$;
- $J(d_i, l)$ a function that indicates whether $d_i$ includes category $l$ (0 or 1).

### 5.3.3. F-Measure

F-Measure [54] conveys the balance between accuracy and diversity. We defined the F-Measure as the harmonic mean of accuracy and diversity:

$$F - measure = \frac{2 * Accuracy * Diversity}{Accuracy + Diversity} \tag{21}$$

As the range of metrics defined in our study are all [0,1], we defined two F-measures for evaluation: the $\boldsymbol{F - nDCG}$ measure as the tradeoff between $\boldsymbol{nDCG}$ and $\boldsymbol{\alpha - nDCG}$ and the $\boldsymbol{F - prec}$ measure as the tradeoff between $\boldsymbol{Prec}$ and $\boldsymbol{Prec - IA}$.

### 5.3. Experimental Results

As previously mentioned, the aim of a recommender system is to generate top-N recommendations for each user in the test set. Since each item has around 2 genres and the total number of genres is less than 30 (See Table 2), it is very likely that a list with a large number of items will cover a large number of genres. So, we set $\mathbf{N}$ to $\mathbf{3}$. Regarding the latent factor dimension, we study the performance of the NMF-based approaches for values: $\mathbf{F} = \{\mathbf{20}, \mathbf{30}, \mathbf{50}, \mathbf{100}\}$. Additionally, we investigate the effect of the size of the top-k diverse neighbors on diversity with $\mathbf{k} = \{\mathbf{20}, \mathbf{30}, \mathbf{50}, \mathbf{100}, \mathbf{150}, \mathbf{200}, \mathbf{300}\}$. Experimental results of different approaches in accuracy, diversity and F-measures on the ML-100k, ML-1M and MT-750k datasets are shown in Figure 2, Figure 2 and Figure 3.

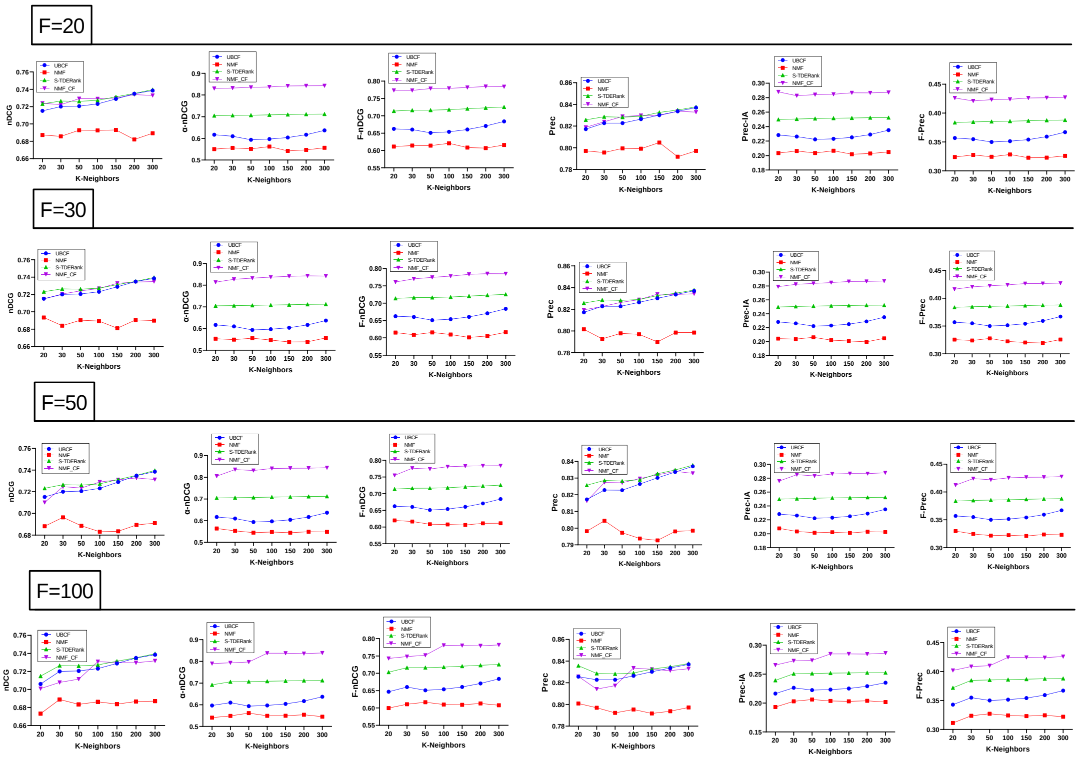


**Figure 2**. Performance on ML-100K Dataset

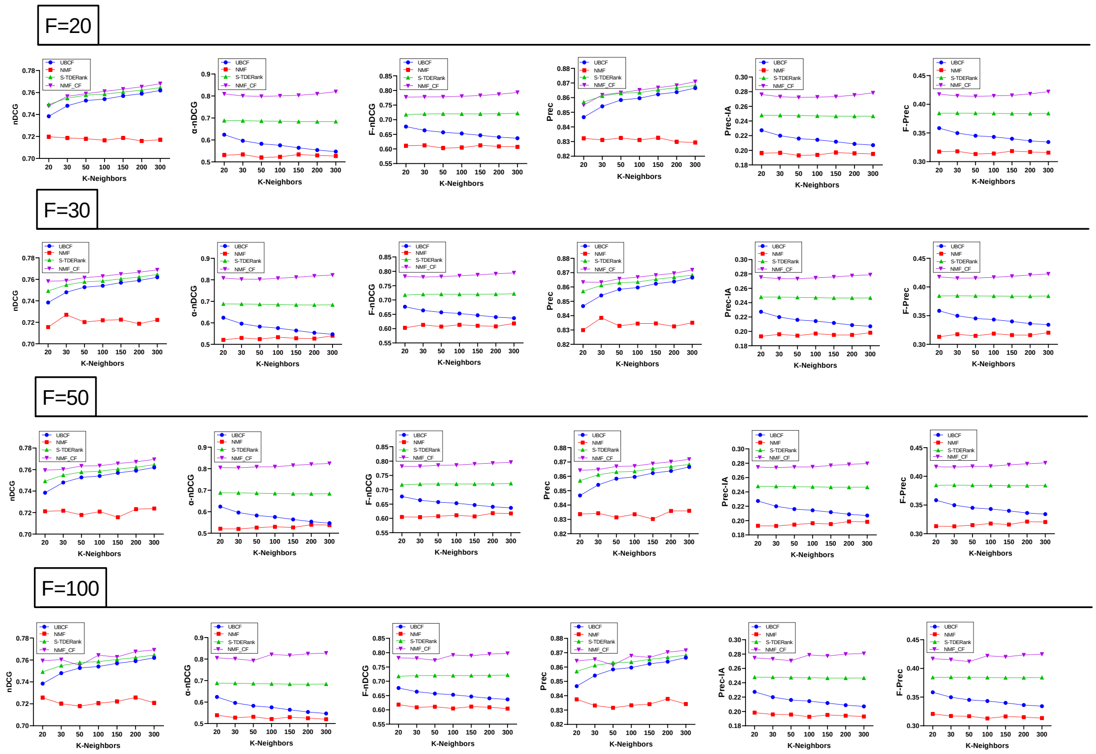


**Figure 3.** Performance on ML-1M Dataset

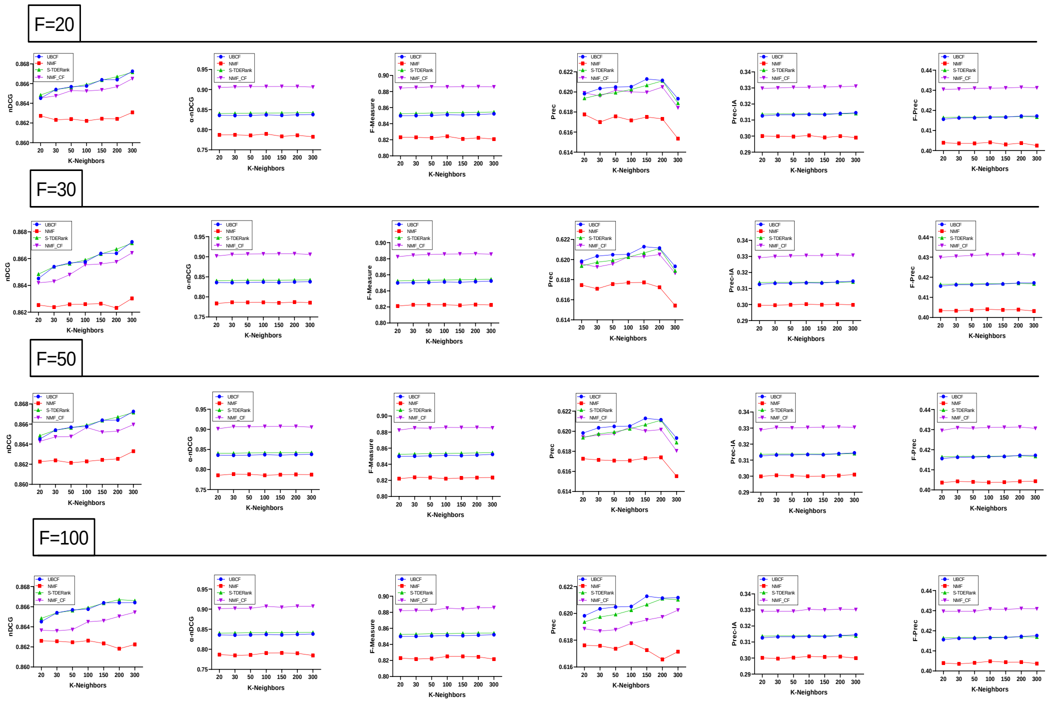


**Figure 4.** Performance on MT-750K Dataset

To summarize the figures above, the Tables belowshows the results of different approaches by averaging different values obtained for each size of neighbors and for $\mathbf{F = 30}$. From Table 3 and Table 4, we can clearly notice the performance and robustness of our approach, which proved a significant improvement on diversity and F-measures. Comparing with the best performing baseline algorithm, we can observe that:

- Our proposed approach has achieved greater improvement on diversity based on $\alpha - \text{nDCG}$ and Prec_IA metrics. NMF-CF has achieved17.6%, 18.2% and 7.6% improvement on $\alpha - \text{nDCG}$ for ML-100k, ML-1M and MT-750k datasets respectively.

- Although generally recommendation algorithms that tend to improve diversity causes a loss on accuracy, while our method has shown its efficiency in maintaining a good level of accuracy. As illustrated, in Table 3, $\mathrm{NMF-CF}$ obtains $0.64\%$ improvement on nDCG for ML-1M dataset with $0.4\%$ and $0.09\%$ loss on nDCG forML-100k andMT-750krespectively. For the precision, the loss was $0.11\%$ for only theMT-750k dataset.
- For the F-measures which represent the trade-off between accuracy and diversity, the advanced NMF-CF technique presents the best compromise on all databasesML-100k, ML-1M and MT-750kwith an improvement of $8\%, 9.1\%, 3.7\%$ respectively on $\mathrm{F-nDCG}$ and $9.6\%, 8.8\%$ and $3.4\%$ improvement on $\mathrm{F-Prec}$.

**Table 2.** Performance Comparison for F=30 on ML-100K, ML-1M and MT-750K datasets in nDCG,α-nDCG and F-ndcg metrics.

| | | UBCF | NMF | S-TDERank | NMF_CF |
|---|---|---|---|---|---|
| **ML-100K** | **nDCG** | 0,7259 | 0,6884 | **0,7299** | 0,7269 |
| | **α-nDCG** | 0,6106 | 0,5483 | 0,7085 | **0,8338** |
| | **F-ndcg** | 0,6632 | 0,6104 | 0,7190 | **0,7767** |
| **ML-1M** | **nDCG** | 0,7530 | 0,7212 | 0,7582 | **0,7631** |
| | **α-nDCG** | 0,5776 | 0,5292 | 0,6853 | **0,8103** |
| | **F-ndcg** | 0,6533 | 0,6104 | 0,7199 | **0,7860** |
| **MT-750K** | **nDCG** | 0,8659 | 0,8626 | **0,8660** | 0,8652 |
| | **α-nDCG** | 0,8363 | 0,7858 | 0,8413 | **0,9060** |
| | **F-ndcg** | 0,8509 | 0,8224 | 0,8535 | **0,8851** |

**Table 3**. Performance Comparison for F=30 on ML-100K, ML-1M and MT-750K datasets in Prec, Prec_IA, F-Prec metrics.

| | | UBCF | NMF | S-TDERank | NMF_CF |
|---|---|---|---|---|---|
| **ML-100K** | **Prec** | 0,8172 | 0,8228 | 0,8227 | **0,8264** |
| | **Prec_IA** | 0,2271 | 0,2031 | 0,2514 | **0,2843** |
| | **F- Prec** | 0,3563 | 0,3237 | 0,3860 | **0,4234** |
| **ML-1M** | **Prec** | 0,8588 | 0,8340 | 0,8636 | **0,8670** |
| | **Prec_IA** | 0,2150 | 0,1955 | 0,2471 | **0,2756** |
| | **F- Prec** | 0,3439 | 0,3167 | 0,3842 | 0,4182 |
| **MT-750K** | **Prec** | **0,6204** | 0,6172 | 0,6200 | 0,6197 |
| | **Prec_IA** | 0,3135 | 0,2999 | 0,3138 | **0,3303** |
| | **F- Prec** | 0,4165 | 0,4036 | 0,4167 | **0,4309** |

## VI. CONCLUSION & FUTURE WORK

In this paper we investigate the trade-off problem between accuracy and diversity in Recommender Systems. We proposed an improved User-based Collaborative filtering approach based on Non-negative matrix factorization along with personalized diversity for each user. The comparison with the state-of-the-art approaches validated the performance of our approach to maintain accuracy while improving diversity. For future work, we will explore others matrix factorization methods for future improvement. In addition, this research can be extended to evaluate other aspects such as recommendation novelty and serendipity. Finally, more baseline diversification methods of recommender systems can be examined along with online experiments.

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